Title: Log On to the Exponential Regression Function

Brief Overview:

The students will derive an exponential regression function from linear regression on (x, log y). They will use the correlation coefficient, residuals, and related linear regressions to determine if a linear or an exponential regression is a better fit for the given set of data.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  Students will develop and compare linear and exponential regression models for sets of real-world data.

- **Mathematics as Communication**
  Students will develop conclusions from their investigations, answer open-ended questions, and express their findings in writing on the given worksheets. Students will also work together in pairs and exchange ideas in order to arrive at solutions for the given problems.

- **Mathematics as Reasoning**
  Students will decide which type of function is a better fit to a set of data based on the linear form of each regression equation.

- **Statistics**
  Students will fit linear and exponential curves to given data sets using the TI-83 calculator. They will compute and analyze residuals.

- **Functions**
  Students will employ the fundamental properties of linear, exponential, and logarithmic functions in their analysis and expression of the regression functions.

Grade/Level:

Grades 9-12

Duration/Length:

This activity is designed to take from 1 to 3 days.

Prerequisite Knowledge:

Students should have working knowledge of the following:

- Basic properties of linear regression and correlation coefficient
- Basic properties of linear, exponential and logarithmic functions
- Basic facility with the TI-83 data entry, statistics and graphing operations

Objectives:

Students will:

- work in pairs to analyze data and solve problems.
- enter and analyze given data using the TI-83.
- discuss the significance of residuals in determining the best fit model.
- use linear regression to develop an exponential function to model the data.
- complete assigned problems and worksheets.
- choose the better of the two regression models and justify their choice.

Materials/Resources/Printed Materials:

- TI-83 calculators
- TI-83 overhead with calculator
- Student Worksheets

Development/Procedures:

- Group students in pairs.
- Complete Linear Regression Worksheet I (1-7).
- Discuss results.
- Introduce residuals and how to find them on the calculator.
- Calculate and plot the residuals on the TI-83 and complete Linear Regression Worksheet I (8-13).
- Discuss the results and why linear regression may not be best model.
- Introduce exponential model.
- Complete Exponential Regression Worksheet II.
- Discuss results and conclusions.
- Assign Nutritional Data Worksheet III for students to work on independently.

Evaluation:

- Worksheet I: The teacher should check this worksheet for completeness. The teacher should then lead whole class discussion emphasizing accuracy of answers and the level of essay expected on discussion questions.

- Worksheet II: The teacher should check this worksheet for completeness. The students should be placed in groups to compare and discuss answers. The teacher may facilitate a whole class student led discussion.
- Worksheet III: This worksheet should be collected and graded by the teacher. Grading should give weight to accuracy of answers, accuracy and neatness of graphs, and thoroughness of discussion answers. Worksheets should then be returned and discussed with students.

Extension/Follow Up:

- Use the linear regression with the log x and y to develop a logarithmic model for a set of data.
- Use the linear regression with the log x and log y to develop a power function model for a set of data.
- Use the TI-83 statistics functions to find exponential, logarithmic and power regressions for given sets of data.
- Use more recent National Debt Data to develop a more current model. (Recommended Website: www.concordcoalition.org)

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Worksheet I
Linear Regression and Residuals

Name____________________________________

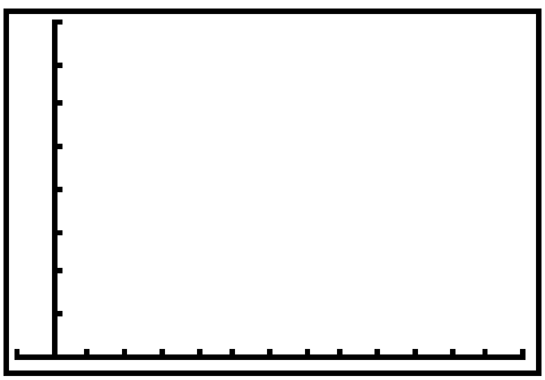
Directions: Use the given set of data, which shows the annual debt in trillions of dollars for the years 1980 to 1991, to complete the exercises on this worksheet.

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991
Debt	.909	.994	1.1	1.4	1.6	1.8	2.1	2.3	2.6	2.9	3.2	3.6

1. Enter the data, using the number of years after 1980 as the entries in list L1 (0=1980, 1=1981, etc.) and the number of trillions of dollars of debt as the entries in list L2.

2. Use the **STATPLOT** feature of the TI-83 calculator to create a scatter plot of the data in Plot1. L1 is X and L2 is Y. Use Zoom 9 to get an appropriate window, then display the scatter plot graph.

3. Copy your scatter plot here. You may need to adjust the window.



4. Find the equation for the line of best fit and the correlation coefficient. On the TI-83, choose **[STAT][CALC] [4]** LinReg (ax+b) **[2^{nd}][1] [,] [2^{nd}][2] [,][Vars]** $\rightarrow$ Y-Vars **[enter] [enter].**

5. Record the linear regression information here:
   a = _________________ b = ___________________ r = ___________________

   regression equation: _________________________

6. Sketch the line on the scatter plot in #3 above.

7. Discuss how well the linear regression model fits the given data.

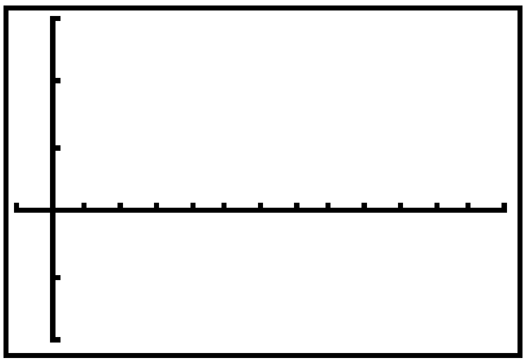
The residual for a data point is the difference between the y-value of the actual data point and the y coordinate of the point on the regression line. We will now compute and plot the residuals for this set of data to learn more about how well the regression line fits the data.

8. To compute the residuals and place them in a list, go to list L3 in the STAT EDIT menu and place the cursor on the L3 title at the top of the list.

9. Enter the formula for computation of the residual, L2-Y1(L1), by typing [2nd] [2] [-] [Vars] $\rightarrow$ Y-Vars [enter][enter][( ] [2nd] [1] [ )] [enter]. The list of residuals will appear in L3.

10. Turn off Plot 1 and use Plot 2 to create a scatter plot with L1 as X and L3 as Y. Use Zoom 9 to obtain an appropriate window.

11. Reproduce your scatter plot here.



12. If the original points were perfectly collinear, what would the graph of the residuals be?

13. Discuss the graph of the residuals and the information that it provides about the goodness of fit of the regression line to the data using the following:
    a. What type of graph did you believe would best fit the original scatter plot (line or curve)? Why?
    b. Does the graph of residuals support your conclusion in part a? Explain.

We will now consider the possibility that an exponential curve is a better fit to this data by investigating the scatter plot of (x, log y) on Worksheet II.

Worksheet II
Linear Regression and Residuals for x, log y

Name: ________________________________

Directions: Use the annual debt data from Worksheet I, as saved in lists L1 and L2, to complete these exercises.

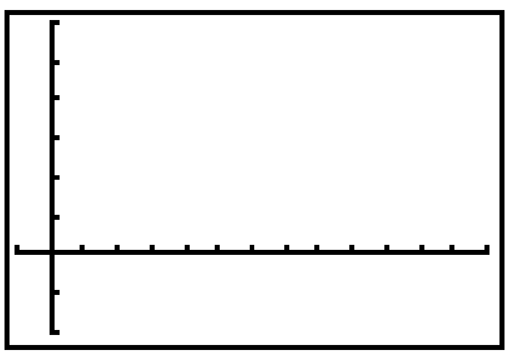
1. Enter the logarithms of each of the Y values in list L4. Place the cursor on the L4 title at the top of the list and type **[log] [2nd][L2][ )] [enter].**

2. Find the linear regression and correlation coefficient for X and log Y using L1 and L4. Store this regression line in Y2. Turn off Y1.

3. Record the regression information here.
   a = __________________ b = __________________ r = __________________
   regression equation: **y =**________________________
   Remember that **y** here is the logarithm of the original Y. (**y** = log Y)

4. Use **PLOT 3** to create a scatter plot of L1 as x and L4 as y. Turn off all other Plots.

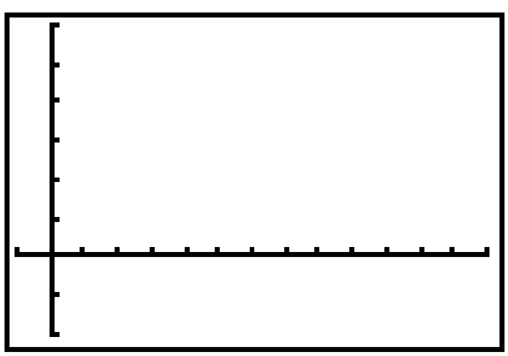
5. Copy both the scatter plot and the regression line here.



6. Compute the residuals and place them in list L5 using the formula L4-Y2(L1).

7. Turn off **PLOT 3** and Y2=. Use **PLOT 2** to make a scatter plot of the residuals, (L1, L5).

8. Reproduce your scatter plot here.



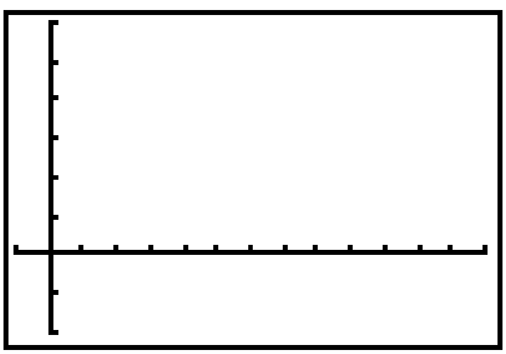
9. Compare the scatter plot of residuals for the X – log Y regression with the scatter plot of the residuals for the X – Y regression. Summarize your conclusions below.

10. The equation of best fit should be expressed as $y = f(x)$. Rewrite the X–log Y regression equation, currently in the form $\log y = ax + b$, in the form $y = f(x)$.

$y =$ _________________________

This equation is called the exponential regression equation or the equation of the exponential curve of best fit for the given data.

11. Enter this new function in Y3. Turn on **PLOT 1**. Turn off all other Plots. Graph **PLOT 1** and Y3 and reproduce the window here.



12. Compare the graph above to the graph of the linear regression function on the scatter plot from #3 of Worksheet I. Summarize your conclusions.

13. Determine whether the linear or the exponential regression is a better model of the given set of data. Justify your conclusion.

Worksheet III
Nutritional Data; Linear or Exponential?

Name___________________________________

Directions: Use the given set of data, which shows the number of calories and the number of grams of fat in certain fast foods, to complete the exercises on this worksheet.

The following data is taken from ***Fast Food Facts*** (1994)

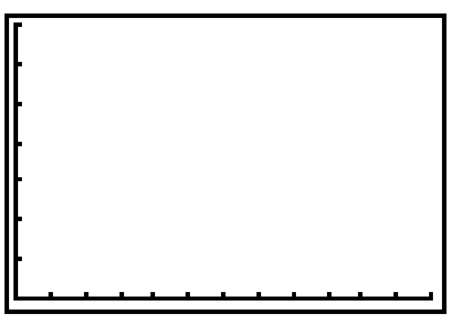
Item	Calories	Grams of Fat
Burger King Whopper	570	31
McDonald’s Big Mac	500	26
Wendy’s Single Hamburger	440	23
Subway 6” Roast Beef	345	12
Hardee’s Roast Beef	380	18
Arby’s Roast Beef	383	18
Hardee’s Fisherman’s Filet	480	21
McDonald’s Filet-O-Fish	370	18
Burger King Ocean Catch	450	28
Kentucky Fried Chicken	284	18
McDonald’s Fried Chicken	270	15
Wendy’s Fried Chicken	280	20
Hardee’s Rise’N Shine Breakfast	320	18
Egg McMuffin	280	11
Burger King Bacon Croissant	353	23

1. Enter the data into your TI-83 calculator. Enter the calories in L1 and the grams of fat in L2.

2. Use the **STAT-CALC to** find the linear regression and correlation coefficient for the relationship between calories (X) and grams of fat (Y), and store the regression in Y1=.

   linear regression_______________________________
   correlation coefficient___________________________

3. Use **PLOT 1** and graph the scatter plot (L1 is X and L2 is Y) and linear regression for this set of data. Copy the graph here.



4. Enter the residuals for this regression in L3.
   Use the calculator and formula: L2-Y1(L1).

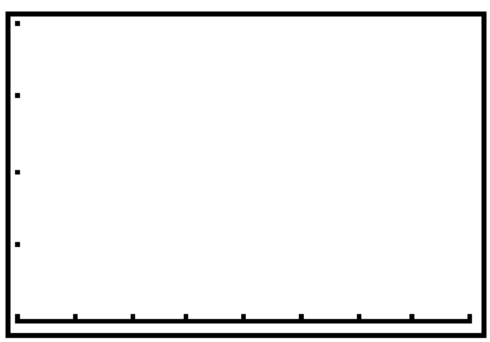
5. Use **PLOT 2** and create the scatter plot of residuals (L1 as X and L3 as Y). Copy the scatter plot here.

6. Enter the logarithms of the values for the grams of fat (L2) into L4. L4=log(L2)

7. Use **STAT-CALC** to find the linear regression and correlation coefficient for the relationship between calories (L1) and log of grams of fat (L4) and store in Y2=.

   linear regression ______________________________
   correlation coefficient __________________________

8. Use **PLOT 1** and graph the scatter plot (use L1 for x and L4 for y) and linear regression of this data. Copy the scatter plot here.



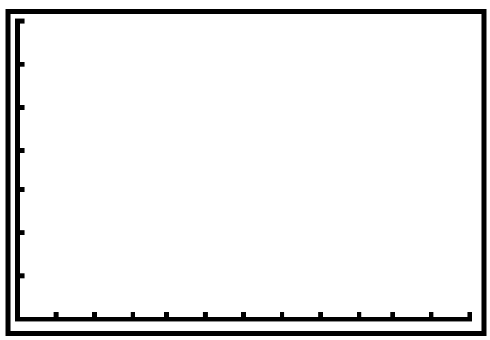
9. Enter the residuals for this linear regression in L5.

10. Use **PLOT 2** and create the scatter plot of these residuals (use L1 as X and L5 as Y). Copy the scatter plot here.

11. Rewrite the X-log Y regression equation in the form y = f(x). Write the equation below and enter it in Y3= on the calculator.

y =________________________________

12. Use **PLOT 1** and create the scatter plot of the original data. Then graph the exponential regression in Y3= on the scatter plot. Copy the scatter plot here.



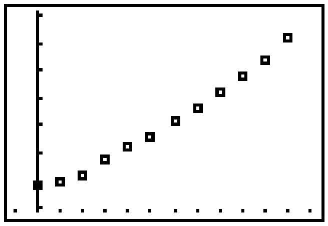
13. Using the results of the previous exercises determine whether the linear or exponential regression is a better model. Justify your answer.

Log On to the Exponential Regression Function
Solution Key

Worksheet I

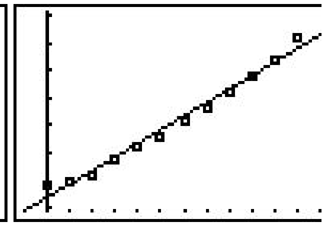
Scatter plot:

```
WINDOW
 Xmin=-1.1
 Xmax=12.1
 Xscl=1
 Ymin=.45153
 Ymax=4.05747
 Yscl=.5
 Xres=1
```



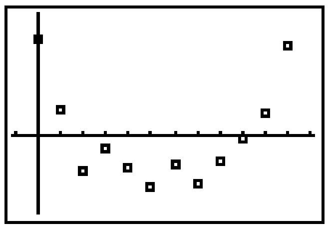
Linear Regression:

```
LinReg
 y=ax+b
 a=.2463461538
 b=.6870128205
 r²=.98378075
 r=.9918572226
```



Residuals:

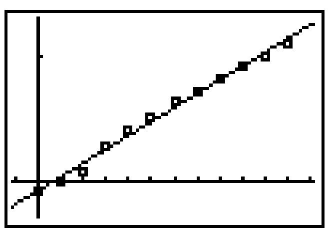
```
WINDOW
 Xmin=-1.1
 Xmax=12.1
 Xscl=1
 Ymin=-.1766678…
 Ymax=.27991141…
 Yscl=.5
 Xres=1
```



Worksheet II

Scatter plot and Linear Regression on X – log Y

```
WINDOW
 Xmin=-1.1
 Xmax=12.1
 Xscl=1
 Ymin=-.1430516…
 Ymax=.65791806…
 Yscl=.5
 Xres=1
```

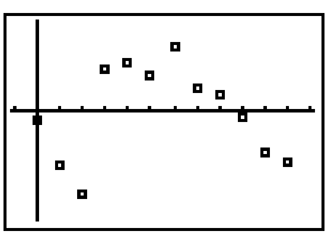


```
LinReg
 y=ax+b
 a=.0558602491
 b=-.0384285006
 r²=.9918130194
 r=.9958980969
```

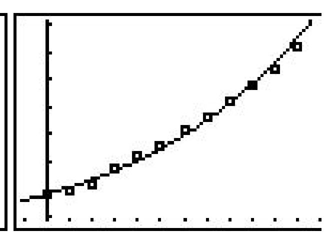
Residuals:

Exponential Regression: $y = 10^{.559x - .0384}$ or $y = \frac{1.137^x}{1.091}$

```
WINDOW
 Xmin=-1.1
 Xmax=12.1
 Xscl=1
 Ymin=-.0416548…
 Ymax=.03524185…
 Yscl=.5
 Xres=1
```



```
WINDOW
 Xmin=-1.1
 Xmax=12.1
 Xscl=1
 Ymin=.45153
 Ymax=4.05747
 Yscl=.5
 Xres=1
```

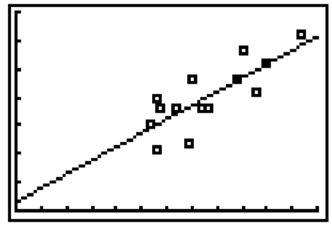


Worksheet III

Linear Regression: y = .0485008565 x + 1.5553507582

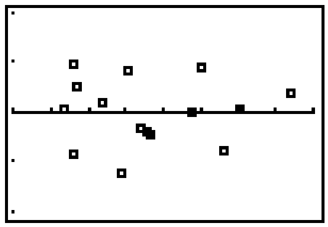
```
LinReg
 y=ax+b
 a=.0485008565
 b=1.553507582
 r²=.6344588785
 r=.7965292703
```

```
WINDOW
 Xmin=0
 Xmax=600
 Xscl=50
 Ymin=0
 Ymax=35
 Yscl=5
 Xres=1
```



Residuals:

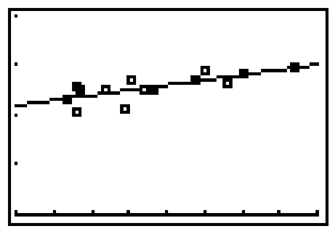
```
WINDOW
 Xmin=200
 Xmax=600
 Xscl=50
 Ymin=-10
 Ymax=10
 Yscl=5
 Xres=1
```



Linear Regression: log y = .0010277703 x + .893986419

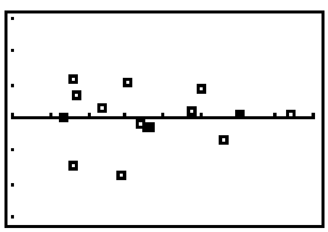
```
LinReg
 y=ax+b
 a=.0010277703
 b=.893986419
 r²=.564098022
 r=.7510645924
```

```
WINDOW
 Xmin=200
 Xmax=600
 Xscl=50
 Ymin=0
 Ymax=2
 Yscl=.5
 Xres=1
```



Residuals:

```
WINDOW
 Xmin=200
 Xmax=600
 Xscl=50
 Ymin=-.3
 Ymax=.3
 Yscl=.1
 Xres=1
```



Exponential Regression Function: $y = 10^{.0010277703x+.893986419}$ or $y = 7.834 \cdot 1.00237^{x}$

```
WINDOW
 Xmin=0
 Xmax=600
 Xscl=50
 Ymin=0
 Ymax=35
 Yscl=5
 Xres=1
```